The Use of Rubrics to Enhance Mathematical Teaching and Learning Practices when Engaging with Challenging Mathematical Tasks

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The planned use of challenging mathematical tasks is explored in this paper. These tasks provide the opportunity for students to improve mathematical thinking by working on problems that they do not yet know how to answer. This research involved a heterogeneous class of Year Three students from a Catholic Parish Primary School in the northern suburbs of Melbourne. A rubric was also developed that was used, in conjunction with these tasks, to support discussions with students, broaden their strategies in finding solutions-and thereby improve their conceptual understanding. These pedagogical approaches were found to support the improvement of both students' conceptual understanding in mathematics and teachers' reflective practice.

This paper examines action research that tested a rubric as a method of identifying different levels of thinking by students working on challenging mathematical tasks. The research explored ways of using challenging tasks to increase the sophistication of students' mathematical strategies and explanations. The study aimed to answer the question "To what extent can rubrics be used to support teachers' use of challenging tasks to broaden the sophistication of students' mathematical concepts?"

This study involved the development and use of a rubric as an instrument to support teachers, students, and other stakeholders in knowing where students are in their mathematical learning and "where to next". The use of rubrics may allow teachers to infer gaps between a student's existing learning and the learning objectives. Francisco and Maher (2011) find that observations made by teachers helps them realise the value of providing students with opportunities to explore ideas and make decisions about their own mathematical reasoning and its development. Teachers need to know how to assess student's reasoning, in addition to assessing mathematical skills but they must be deliberate in the choice of tasks that ask students to apply both reasoning and mathematics skills.

Research evidence supports the use of challenging tasks in developing students mathematical reasoning skills. Clark and Clark (2002) recognise four qualities that characterise such tasks: they must enable students to produce different solutions, use different strategies, offer diverse final presentations, and fully engage students. Boaler (2019) argues that number sense, which is fundamentally important for students to learn, includes the learning of mathematical facts along with a deep understanding of numbers and the ways they relate to each other. Cheeseman et al. (2013) reflect on practices teachers use with these tasks and identify that students can engage with important mathematical ideas, be encouraged to explain their strategies, justify their thinking, and extend their knowledge in new ways.

Teachers must also consider a range of pedagogical factors when selecting challenging tasks. Kirschner et al. (2006) advocate for guided instruction being superior to minimal or even no guidance with challenging tasks. However, Marshall and Horton, (2011) alongside Russo and Hopkins (2017) agree that the value of exploring concepts, before any direct instruction, is in realising students' abilities to reason and think critically. Sullivan et al. (2012) also acknowledge the importance of matching tasks to curriculum content when selecting tasks. Cheeseman et al. (2016) discuss the importance of how a teacher introduces a task, including preparing students to have persistence, connecting the task with student experiences, providing manipulatives, and clarifying the task without showing how to reach the solution. This paper explores an approach aligned with the views of Russo and Hopkins (2017).

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Choosing tasks, structuring lessons around them and then incorporating them successfully into a thematic program of work requires careful thought. Teachers may need support in developing a classroom culture which supports this style of learning (Sullivan et al., 2013). Teachers can, therefore, be disinclined to use challenging tasks (Cheeseman et al., 2013) because they view them as unclear, too demanding or are concerned about low attaining students. Clarke et al. (2014) highlight the significance of a teacher's interest in a task in influencing its success, as well as teacher confidence in the enthusiasm and ability of their students. Kirschner et al. (2006) suggest that minimising instruction may lead to misunderstandings or piecemeal knowledge, so a teacher's approach needs to be balanced against providing too much information to reduce the level of challenge within the task. Jacobs et al. (2014) notes a danger of teachers taking over student thinking, controlling available tools and asking closed questions, removing the agency of students in their learning and development of conceptual understanding. Simon (2017) argues that an understanding of mathematical concepts requires students to learn concepts through mathematical activities in the form of challenging tasks. Rather than students using a sequence of actions already available to them based on their prior knowledge, challenging mathematical tasks support students to build new knowledge. The current study looks at supporting teachers to give sufficient, but not excessive instruction, using an assessment rubric to provide appropriate, timely feedback to students that progresses their conceptual mathematical understanding.

The rubric used for this study was developed from Bloom's Taxonomy (Krathwohl, 2002) and Webb's (1997) Depth of Knowledge Framework. Webb (1997) suggests that "challenge" in learning tasks promotes growth by keeping students engaged and his framework describes the quality of student thinking in various tasks. Krathwohl's update of Bloom's Taxonomy (2002) describes the cognitive level students demonstrate during learning, while the Depth of Knowledge (DoK) focuses more on the context—in this case the challenging task. While Hess et al. (2009) identify some limitations with Bloom's Taxonomy, the current study incorporates both Bloom's and Webb's models into the rubric.

Project Design

Participants

The participants were members of a heterogeneous Year Three classroom in a Catholic Parish Primary School in the northern part of Melbourne. The researcher was the full-time teacher of the class. A Pre-Service Teacher (PST) was also working full time in the classroom at the time of the study and was involved in the data collection. There was no requirement for a selection process as a convenience sample was being used.

Method and Rationale

This study used an instrumental case study approach alongside action research. The action research aspect addressed the need to improve practices and the instrumental case study approach aligned with the observation of a situation. Such approaches provide teachers with opportunities to apply research methods to their teaching (Mills et al., 2010). They can also improve teachers' understanding of classroom practices and raise awareness of student learning that requires further investigation. Teachers can test approaches that may transfer well to similar classrooms (Yin, 2014) and integrate assessments generated by their own research into practice.

The current study used two tasks designed by Russo (2006, a and b): The Doughnut Tree task (which explores exponential doubling) and The Big (not so) Friendly Giant task (which explores halving). The first task was chosen because the class had been working on multiplication using doubling. Russo (2016a) contends that students working in middle primary classrooms are expected to have developed fluency with their doubles facts and should be exploring doubling as a rule. He argues that students would benefit from exploring exponential doubling at a younger age. The second task was chosen to meet the needs of a diverse group of students both providing accessibility and extension using enabling and extending prompts. Both tasks address the mathematically related skill of doubling and its inverse, halving, supporting students to link these two concepts. These tasks were conducted, and data collected from one class in Term 3 of 2018.

Enabling prompts are an integral aspect of challenging task design as they reduce the level of challenge through simplifying the problem, changing how the problem is represented, helping the students connect the problem to prior learning and/or removing a step in the problem (Sullivan, et al., 2006). Examples include reducing the starting numbers for the tasks, providing concrete materials, reducing the number of steps and altering the task presentation expectations.

Extending prompts can be used to engage students who finish the main challenge and may expose students to an additional task that is more challenging, but still requires them to use similar mathematical reasoning, conceptualisations, and representations as the main task (Sullivan et al., 2006). The appropriate prompt is selected by the teacher in real time, developed from their analysis of the potential task difficulties based on perceived cognitive load.

Russo and Hopkins (2017), reflecting on cognitive load theory (Sweller, 1998), identify seven steps to produce challenging mathematical tasks that aim to optimise the cognitive load for each student. These steps are: identify the primary learning objective, develop the task, look for possible other learning objectives, sort any objectives in line with their cognitive load, redesign the task, develop prompts to optimise the cognitive load and propose a lesson summary.

A launch, explore, discuss model (Stein et al., 2008) was used to deliver the lessons. This facilitated more explicit explanations, scaffolded connections and highlighted big mathematical ideas. In the launch phase, the word "challenging" was discussed with the students and the word was defined to engage the students in characterising an appropriate mindset. Each problem was introduced in a separate lesson, alongside available materials and recording expectations.

In the explore phase, students worked on the task individually or in pairs. Students were supported in solving the problem in whichever way suited them. Enabling prompts were offered and students had access to counters, number lines, 100 squares, notes about doubling and halving and were able to ask clarifying questions. Whilst students were working on the solutions three main questions were asked: How would you describe the problem in your own words? Would it help to create a diagram, draw a picture, or make a table? Could you try it with different numbers?

In the discuss phase, the teacher presented a summary of what had been observed, referring to specific strategies used by students, some of whom shared their thinking with the class. After looking at work from the first task, it was noted that although students were solving the problem, their thinking was not clearly shown. Support to assist this was provided in the launch phase for the second task.

In response to observations and discussion with the PST, a follow up lesson was proposed based on discussing ways of presenting strategies and solutions, answering questions with detail, revising, and editing work and explaining mathematical reasoning. It was felt that the students required more explicit teaching. alongside detailed examples of possible ways of presenting their solutions and reasoning. The lesson was based on a simpler task 'What Else Belongs' involving students identifying connections between three numbers–30, 12, 18. The students were asked to look for relationships between the numbers and why they might be placed in a group and then discover other numbers which could also belong in that group.

Student Work

Student work was assessed against criteria from the rubric rather than being competitively ranked. This approach provides students feedback regarding how to improve rather than how they compare with others. A process of moderation was undertaken in which work was de-identified by the PST and shared between a team of three: the researcher, the Numeracy Leader, and the PST, to provide inter-relater reliability. Any work pieces with which scoring was uncertain were placed together for further consideration by another member of the team. If there was still uncertainty the whole team would look at the work. For the purposes of moderation, each team member selected a sample piece for each level of thinking using the DoK stages, and these were compared. For this study, due to ethical requirements, no student work could be reported or presented.

Limitations of the Data Collection

Student work was collected at the end of each teaching session by the PST to maintain as much anonymity as possible. Apart from recognising some handwriting the researcher was not aware as to who had completed which work samples. According to Fraser (1997) the concept of a teacher as a researcher enables credible educational research to be undertaken, but the ethical predicaments faced could be more challenging than those met by an external researcher.

Data Sources and Analysis

The primary data for analysis was generated by scoring student performances using rubrics and was based on expert teacher evaluation of student responses and explanations of their thinking during challenging tasks and student work (artefacts). Artefacts were grouped in terms of similar approaches to the task. These were rated against the rubrics. Scores were then categorised according to the nature of the students' responses. The final analysis involved a final sample size of 15 students selected randomly from those who had completed both tasks.

The approaches used by students reflected varying degrees of sophistication in their application of mathematical strategies. Approaches included the use of drawings, number lines, repeated addition and subtraction, formal algorithms, partitioning numbers to make doubling/halving simple, as well as the direct use of multiplication and division. The use of drawings was the most common strategy followed by partitioning numbers. The use of number lines and repeated addition and subtraction occurred with equal frequency. The direct use of multiplication facts and division was rare in both tasks.

Analysis

The collected work was assessed against the developed rubric, as shown in Table 1. Responses were scored from 0 to 4 according to the levels on the rubric. Table 2 shows the types of thinking—Problem Solving, Reasoning, Representation and Connection—with a Zone of Proximal Development (ZPD) (Vygotsky & Cole) for each task. This ZPD enables the teacher to see areas of growth and where further support is needed.

Table 1

Rubric For Levels of Thinking Used in Exploring Challenging Mathematical Tasks

Level of Thinking	Problem Solving	Reasoning	Representation	Connection
Level 1 Recall and reproduction	Did not understand the task. What did the student appear to interpret the task as entailing?	Mathematical thinking is incorrect.	Used no mathematical language and/or notation. Diagrams not relating to task.	Did not make any connections to the task or the numbers in the task.
Level 2 Basic application of skills and concepts	Understood part of the task. Needed help to understand the entire task. Strategy works for part of the task.	Some mathematical thinking or explanation is correct. Needed help to explain the task.	Used some mathematical language and/or notation. Some diagrams were used to represent the task.	Tried to make connections to previous learning related to the task.
Level 3 Strategic thinking	Understood the task and the strategy they used works.	Mathematical thinking and explanation correct. Some thinking systematic.	Used clear mathematical language and/or notation Diagrams related to task.	Made some mathematical connections to previous
Level 4 Extended Thinking	Understood the task. Used an efficient strategy. Extension activities were completed.	Detailed and accurate explanation of the strategy used to solve the task. Mathematical thinking was correct and systematic.	Used specific math language and/or notation throughout their work. Diagrams related directly to task and explained student thinking. Extension activities were completed.	Recorded mathematical connections to mathematical big ideas and strategies previously used. Extension activities were completed.

It appears that in Task 1 students had more problems with Representation whereas in Task 2 Connections was more of an issue. Scores for three areas—Problem Solving, Reasoning and Representation—increased quite considerably, whereas the Connection score remained very similar. It appears this was an area with which many students struggled and where future explicit teaching needs to be focused. Problem Solving scores had the greatest increase. This could be attributed to greater familiarity with the type of tasks, students acting on discussions and feedback from the first task and/or students finding the second task easier to solve.

Table 2

Conditional Formatting of Rubric Elements for Analysed Work Samples to Create A ZPD

	Task 1. The Doughnut Tree				Task 2. The Big (not so) Friendly Giant			
	Problem Solving	Reasoning	Representation	Connection	Problem Solving	Reasoning	Representation	Connection
22	1	1	1	1	1	1	1	1
16	1	1	1	1	2	2	2	1
19	1	2	1	1	2	2	2	1

	Task 1. The Doughnut Tree				Task 2. The Big (not so) Friendly Giant			
	Problem Solving	Reasoning	Representation	Connection	Problem Solving	Reasoning	Representation	Connection
23	1	2	2	2	2	1	1	1
21	1	1	1	1	3	3	3	1
5	1	1	1	2	3	3	2	1
27	2	2	2	2	2	2	2	1
12	2	2	1	1	3	3	3	1
6	2	2	1	1	4	3	2	2
10	2	2	2	2	4	3	2	1
8	2	2	2	2	3	3	3	2
26	2	2	2	2	3	3	3	2
28	3	2	2	2	3	3	3	3
11	3	2	2	2	4	3	4	4
24	3	3	2	1	4	4	4	3

Results and Evaluation

The data analysis and results were related back to the initial research question proposed: "To what extent can rubrics be used to support teachers' use of challenging tasks to broaden the sophistication of students' mathematical concepts?" Concepts analysed included Problem solving, Representation and Reasoning. Results from the ZPD analysis showed several interesting findings. Nearly all students used a broader range of strategies in the second task compared with the first. Students who used broader strategies used them at a more sophisticated level. Most students diversified the way they represented their thinking. About half of the students showed limited development in Connection, indicating an area for targeting teaching strategies. That students were least successful with Connection confirmed the teachers' view that previous tasks these students have experienced have focused on solving the problem rather than conceptual understanding. The element of Connection scored low in both tasks. This may have been because these tasks were different from other mathematics tasks being undertaken and students viewed them in isolation from their everyday mathematics lessons.

Conclusion

This research was undertaken hoping to identify different levels of thinking by students working on challenging tasks. With all classroom-based research, uncertainty in data is likely. There are limitations of this being a small study as it was conducted in one classroom of one school, over a short period of time. For these reasons any claims cannot be generalised. However, the findings suggest there are many types of thinking demonstrated by students working on challenging tasks. This study suggests that it is possible to assess the depth of thinking that students engage in when solving challenging tasks. It appears that teachers can support students in refining their thinking and their ability to record strategies and reasoning.

There are implications for the classroom. These include the potential of teacher developed rubrics that support observation and timely feedback. Feedback may move students on from their current level of conceptual understanding, broaden the range of strategies they are comfortable in using and encourage clear explanations of thinking. Teachers could develop a progression of strategies towards a conceptual understanding of multiplication and division incorporating what they observe in their students.

This small study could be used to inform a larger study around using evidence informed practice and formative assessments to improve teaching and learning for a range of different mathematical concepts. Further research could include refining the descriptive sections of the rubric and developing challenging tasks for use in other areas of mathematics. Consideration would need to be taken of other elements involved in the successful implementation of challenging tasks including encouraging students to persist, fostering students in the skills of listening to others and teacher reflections on their own and student experiences. Another study, published after this research was completed, produced an Assessing Mathematical Reasoning Rubric (Loong et al., 2018). This work creates a rubric which teachers found helpful to support their understanding of how to develop reasoning in students and in reporting student progress. As with the study by Loong et al. (2018) further research is needed to ensure that such rubrics are pragmatic, time efficient and provide appropriate information.

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